Towards a model independent approach to fragmentation functions

Ekaterina Christova[1] **and Elliot Leader**[2]

[1] *Institute for Nuclear Research and Nuclear Energy, Sofia*
[2] *Imperial College, London*

We show that the difference cross sections in unpolarized SIDIS $e+N \to e+h+X$ and pp hadron production $p+p \to h+X$ determine independently in a model independent way, in any order in QCD, the two FFs: $D_u^{h-\bar{h}}$ and $D_d^{h-\bar{h}}$, $h=\pi^\pm, K^\pm$ or a sum over charged hadrons. If both $K^\pm$ and K_s^0 are measured, then $e^+e^- \to K+X$, $e+N \to e+K+X$ and $p+p \to K+X$ present independent measurements of just one FF: $D_{u-d}^{K^++K^-}$. The above results allow to test the existing parametrizations, obtained with various different assumptions about the FFs, and to test the Q^2 evolution and factorization.

1 Introduction

There is at present great interest in learning how the spin of the nucleon is built up from the angular momentum of its constituents. A key ingredient in this is a knowledge of the polarized parton densities. Most of our knowledge of the polarized PDFs comes from inclusive deep inelastic scattering (DIS), where, however, one obtains information only on the combinations $\Delta q(x) + \Delta\bar{q}(x)$. Information on the polarized sea quark densities can, in principle, be obtained from semi-inclusive deep inelastic scattering (SIDIS) reactions of the type $l+N \to l+h+X$, and from semi-inclusive hadron-hadron reactions like $p+p \to h+X$. However both of the latter require a knowledge of parton fragmentation functions (FFs) describing the transition $parton \to h+X$.

These are not very well known, being obtained principally from analyses of $e^+ + e^- \to h^\pm + X$, where, however, only the combinations $D_q^h + D_{\bar{q}}^h = D_q^h + D_q^{\bar{h}} \equiv D_q^{h+\bar{h}}$ occur. Several sets of FFs are available in the literature – Kretzer [1], KKP [2], AKK [3]), HKNS [4] etc. One study in [5] combined e^+e^- data with unpolarized SIDIS data on $\pi^\pm$ production, in [6] a combined analysis of e^+e^- and $pp(\bar{p})$ data was carried on, and quite recently, for the first time, fragmentation functions were extracted from a global fit to e^+e^-, SIDIS and $pp \to \pi^\pm X$ data [7]. A comprehensive review on the current status of the fragmentation functions is presented in [8].

Two points should be noted: 1) in all of these analyses (except [5]) it was necessary to impose some relations, based on theoretical prejudice, between different

FFs[1], and 2) there is significant disagreement between the various analyses for some FFs.

For these reasons it is important to try to find ways of extracting FFs without any theoretical assumptions about relations between them. In this paper we show how information on certain combinations of FFs can be obtained in a model independent way from both unpolarized SIDIS and semi-inclusive *pp* reactions. The key experimental ingredients are the differences between cross-sections for producing hadrons and producing their antiparticles i.e data on $d\sigma^{h-\bar{h}} \equiv d\sigma^h - d\sigma^{\bar{h}}$. We are informed that precise data on such observables is feasible [9].

Our expressions below correspond to an NLO treatment. LO expressions can be obtained by putting $\alpha_s = 0$, replacing convolutions by ordinary products and using only LO formulae for the partonic cross-sections.

2 The cross-sections differences

In this section we consider the cross-sections differences for the two semi-inclusive processes with charged hadrons $h^\pm$:

$$e + N \to e + h^\pm + X \quad \text{and} \quad p + p \to h^\pm + X \tag{1}$$

and define

$$\sigma_N^{h^+-h^-} \equiv \sigma_N^{h^+} - \sigma_N^{h^-}, \quad N = p, d \quad \text{and} \quad \sigma_{pp}^{h^+-h^-} \equiv \sigma_{pp}^{h^+} - \sigma_{pp}^{h^-}. \tag{2}$$

Using C-invariance of the strong interactions

$$D_g^{h^+-h^-} = 0, \qquad D_{\bar{q}}^{h^+-h^-} = -D_q^{h^+-h^-} \tag{3}$$

we obtain rather simple expressions for the cross-section differences and show that in any order of QCD they are expressed in terms of only non-singlet (NS) combinations of the FFs.

2.1 Unpolarized SIDIS

For unpolarized $e + N \to e + h^\pm + X$ we obtain [2]:

$$d\sigma_p^{h^+-h^-}(x, z, Q^2) = \frac{1}{9}\left[4u_V \otimes D_u + d_V \otimes D_d + s_V \otimes D_s\right]^{h^+-h^-} \otimes \left(1 + \frac{\alpha_s}{2\pi} C_{qq}\right) \tag{4}$$

$$d\sigma_d^{h^+-h^-}(x, z, Q^2) = \frac{1}{9}\left[(u_V + d_V) \otimes (4D_u + D_d) + 2s_V \otimes D_s\right]^{h^+-h^-} \otimes \left(1 + \frac{\alpha_s}{2\pi} C_{qq}\right). \tag{5}$$

[1] Though in AKK almost no relations were imposed, we doubt that data is enough to determine *all* FFs independently.

[2] In our formula for the SIDIS cross sections, the common kinematic factors have been omitted, see [10] for the complete expressions.

where u_V and d_V are the usual valence quarks

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \text{and we define} \quad s_V = s - \bar{s}\cdot \tag{6}$$

Here x, z, Q^2 are the usual deep inelastic kinematic variables: $x = Q^2/2P.q = Q^2/2M\nu$, $z = P.P^h/P.q = E^h/\nu$, E and E^h are the Lab energies of the incoming lepton and the final hadron, and the C_{qq} are Wilson coefficients. Note that: 1) $\sigma_N^{h^+-h^-}$ depends only on NS PDFs and FFs and is independent of the less well known gluon quantities $g(x)$ and D_g^h, and 2) the FFs enter multiplied by $q_V = q - \bar{q}$ which implies that the contributions of D_u^h and D_d^h are enhanced by the large valence quark densities, while D_s^h is suppressed by the small factor $(s-\bar{s})$. Recently a strong bound on $(s-\bar{s})$ was obtained from neutrino experiments, $|s-\bar{s}| \leq 0.025$ [11]. This implies that the large uncertainties in D_s^h should not affect strongly the results for $\sigma_N^{h^+-h^-}$, and we expect the contribution of $(s-\bar{s})D_s^{h^+-h^-}$ to be within the experimental error and to be neglegible. Then Eqs. (4) and (5) provide two independent measurements for the two unknown FFs $D_u^{h^+-h^-}$ and $D_d^{h^+-h^-}$. These equations hold for the sum over charged hadrons and for each identified hadron separately.

Further information can be obtained when the final hadrons are identified.

1) If $h = \pi^\pm$ then Eqs. (4) and (5) present two independent measurements that determine $D_u^{\pi^+-\pi^-}$ and $D_d^{\pi^+-\pi^-}$ in a model independent way. Comparison to the existing parametrizations for $D_u^{\pi^\pm}$ and $D_d^{\pi^\pm}$ would be a check of these parametrizations. Also, comparing $D_u^{\pi^+-\pi^-}$ and $D_d^{\pi^+-\pi^-}$ would check the usually made assumption

$$D_u^{\pi^+-\pi^-} = -D_d^{\pi^+-\pi^-}. \tag{7}$$

Recently in [7] it was shown, for the first time, that this relation might be violated up to 10 %.

If eq. (7) holds, then Eqs. (4) and (5) look particularly simple:

$$d\sigma_p^{\pi^+-\pi^-} = \frac{1}{9}\left[4u_V - d_V\right] \otimes \left(1 + \frac{\alpha_s}{2\pi}C_{qq}\right) \otimes D_u^{\pi^+-\pi^-} \tag{8}$$

$$d\sigma_d^{\pi^+-\pi^-} = \frac{1}{3}\left[u_V + d_V\right] \otimes \left(1 + \frac{\alpha_s}{2\pi}C_{qq}\right) \otimes D_u^{\pi^+-\pi^-}. \tag{9}$$

Thus, if (7) holds, it should be possible to express $\sigma_p^{\pi^+-\pi^-}$ and $\sigma_d^{\pi^+-\pi^-}$ in terms solely of a single quantity, $D_u^{\pi^+-\pi^-}$.

2) If $h = K^\pm$ then Eqs. (4) and (5) determine in a model independent way $D_u^{K^+-K^-}$ and $D_d^{K^+-K^-}$. Comparing $D_u^{K^+-K^-}$ to the existing parametrizations would check the parametrizations for kaon FFs. In all parametrizations it is always assumed that

$$D_d^{K^+-K^-} = 0. \tag{10}$$

The above measurement would be a direct test of this assumption.

If (10) holds, Eqs. (4) and (5) look particularly simple:

$$d\sigma_p^{K^+-K^-} = \frac{4}{9} u_V \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) \otimes D_u^{K^+-K^-} \tag{11}$$

$$d\sigma_d^{K^+-K^-} = \frac{4}{9} [u_V + d_V] \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) \otimes D_u^{K^+-K^-}. \tag{12}$$

Thus, if (10) holds, without any other assumptions, $\sigma_p^{K^+-K^-}$ and $\sigma_d^{K^+-K^-}$ will be determined by a single FF, $D_u^{K^+-K^-}$.

Recently, very precise HERMES data on charged pion and kaon production on unpolarized SIDIS were presented in [9]. This would allow to construct the discussed cross-section differences with enough precision.

2.2 Unpolarized semi-inclusive hadron-hadron reactions

According to the factorization theorem, the general expression for single inclusive production of a hadron h with high transverse momentum in proton-proton collisions

$$p\,(P_A) + p\,(P_B) \to h\,(P^h) + X \tag{13}$$

is given by

$$\begin{aligned} E^h \frac{d\sigma_{pp}^h}{d^3P^h} &= \sum_{a,b,c} \int dx_a \int dx_b \int dz f_a^A(x_a, \mu_F) f_b^B(x_b, \mu_F) D_c^h(z, \mu_F') \times \\ &\times d\hat{\sigma}_{ab}^{cX}(x_a P_A, x_b P_B, P^h/z, \mu_R, \mu_F, \mu_F'). \end{aligned} \tag{14}$$

Here the sum is over all contributing partonic channels $a + b \to c + X$ and $d\hat{\sigma}_{ab}^{cX}$ are the corresponding partonic cross sections [see Eq. (20)] calculable in perturbative QCD; μ_F, μ_F' and μ_R are the factorization scales associated with the quark densities, fragmentation functions and renormalization respectively, which, in the following, we take as equal.

Using C-invariance, Eq. (3), without any assumptions about FFs and PDFs, we obtain the following expression for the cross-section differences valid in any order in QCD:

$$\begin{aligned} E^h \frac{d\sigma_{pp}^{h^+-h^-}}{d^3P^h} &= \frac{1}{\pi} \int dx_a \int dx_b \int \frac{dz}{z} \times \\ &\times \sum_{q=u,d,s} [L_q(x_b, t, u) q_V(x_a) + L_q(x_a, u, t) q_V(x_b)]\, D_q^{h^+-h^-}(z) \end{aligned} \tag{15}$$

where we have neglected contributions from heavy quarks since their contributions are proportional to $c-\bar{c}$, $b-\bar{b}$, $t-\bar{t}$ respectively[3]

Here

$$L_u(x,t,u) = \tilde{u}(x)\,d\hat{\Sigma}(s,t,u) + [\tilde{d}(x)+\tilde{s}(x)]\,d\hat{\sigma}_{qq'}^{qX}(s,t,u) + g(x)\,d\hat{\sigma}_{qg}^{(q-\bar{q})X}(s,t,u) \quad (16)$$
$$L_d(x,t,u) = \tilde{d}(x)\,d\hat{\Sigma}(s,t,u) + [\tilde{u}(x)+\tilde{s}(x)]\,d\hat{\sigma}_{qq'}^{qX}(s,t,u) + g(x)\,d\hat{\sigma}_{qg}^{(q-\bar{q})X}(s,t,u) \quad (17)$$
$$L_s(x,t,u) = \tilde{s}(x)\,d\hat{\Sigma}(s,t,u) + [\tilde{u}(x)+\tilde{d}(x)]\,d\hat{\sigma}_{qq'}^{qX}(s,t,u) + g(x)\,d\hat{\sigma}_{qg}^{(q-\bar{q})X}(s,t,u) \quad (18)$$

where

$$d\hat{\Sigma} \equiv \left[d\hat{\sigma}_{qq}^{qX}(s,t,u) + \frac{1}{2}\,d\hat{\sigma}_{q\bar{q}}^{(q-\bar{q})X}(s,t,u)\right] \qquad \tilde{q} \equiv q+\bar{q} \quad (19)$$

The partonic cross-section $d\hat{\sigma}_{ab}^{cX}$ for the inclusive process $a+b\to c+X$ is a function of the corresponding Mandelstam variables:

$$\begin{aligned} &d\hat{\sigma}_{ab}^{cX}(s,t,u) \equiv \frac{d\hat{\sigma}}{dt}(ab\to cX), \quad s=(p_a+p_b)^2=(x_aP_A+x_bP_B)^2,\\ &t=(p_a-p_c)^2=(x_aP_A-p_c)^2, \quad u=(p_b-p_c)^2=(x_bP_B-p_c)^2,\\ &\quad p_c = P^h/z, \end{aligned} \quad (20)$$

where P^h stands for $P^{h^\pm}$ respectively. The $d\hat{\sigma}_{ab}^{cX}$ are calculated in perturbative QCD. In LO these are $2\to 2$ QCD scattering processes, i.e. X stands just for one parton, $s+t+u=0$ and one of the integrations can be done immediately. In total there are 8 different LO cross sections, expressions for which can be found in many places, for example [14]. In NLO, apart from the virtual one-loop corrections to the $2\to 2$ processes, also real $2\to 3$ new processes of order $O(\alpha_s^3)$ are included. This leads to twenty different inclusive processes [15], [16]. In this case s, t and u are independent variables. Eq. (15) implies that due to C-invariance only four inclusive partonic cross sections contribute to $d\sigma_{pp}^{h^+-h^-}$ in LO and six in NLO. These are:

$$\begin{aligned} LO: &\quad qq'\to q(q'), \quad qq\to q(q), \quad q\bar{q}\to q(\bar{q}), \quad qg\to q(g)\\ NLO: &\quad qq'\to qX, \quad qq\to qX, \quad q\bar{q}\to qX,\bar{q}X \quad qg\to qX,\bar{q}X \end{aligned} \quad (21)$$

where the final q or $\bar{q}$ are the fragmenting quarks.

The cross section (15) involves only NS FFs and, thus, the most troublesome D_g^h does not contribute. Also, we would like to emphasize that the structure of the cross section is just the same as in SIDIS — $D_q^{h^+-h^-}$ enters always multiplied by $(q-\bar{q})=q_V$, i.e. in the combination $q_V\,D_q^{h^+-h^-}$. This implies again that $D_u^{h^+-h^-}$ and $D_d^{h^+-h^-}$ are enhanced by the large valence-quark densities, while $D_s^{h^+-h^-}$ is

[3]Strictly speaking, in NLO eq.(15) is an exact expression only if the masses of quarks are negligible, $m_q/\sqrt{s}\ll 1$. For heavy flavour production a charge asymmetry of order α_s^3 is generated [12, 13] and more partonic processes will contribute. However, these effects seem too small to affect our considerations.

suppressed by the small quantity $(s-\bar{s})$ and its contribution should be negligible. However, that should be checked by calculating $L_q(s,t,u)$, which depends only on known quantities.

Thus, ep, ed and pp semi-inclusive cross-section differences determine the same combinations of FFs: $D_u^{h^+-h^-}$, $D_d^{h^+-h^-}$ and $D_s^{h^+-h^-}$. Using the experimentally well justified approximation $s=\bar{s}$, $D_s^{h^+-h^-}$ will not contribute and the three semi-inclusive difference cross sections of ep, ed and pp scattering determine independently the two quantities $D_u^{h^+-h^-}$ and $D_d^{h^+-h^-}$. If $s-\bar{s}=0$, eq. (15) reads:

$$\begin{aligned} E^h\frac{d\sigma_{pp}^{h^+-h^-}}{d^3P^h} &= \frac{1}{\pi}\int dx_a \int dx_b \int \frac{dz}{z} \times \\ &\times \Big\{ [L_u(x_b,t,u)u_V(x_a) + (x_a \leftrightarrow x_b, u \leftrightarrow t)]\, D_u^{h^+-h^-}(z) + \\ &\quad + [L_d(x_b,t,u)d_V(x_a) + (x_a \leftrightarrow x_b, u \leftrightarrow t)]\, D_d^{h^+-h^-}(z) \Big\}. \qquad (22) \end{aligned}$$

As $D_q^{h^+-h^-}$ are non-singlets the gluon FF that introduces, in general, lot of uncertainties will not appear in the Q^2-evolution.

Note that taking $s-\bar{s}=0$ is not an assumption – it is an approximation linked to the precision of the experiment. If the accuracy for the cross-section differences justifies doing so, the strange quark contribution can be included and the cross-section differences will provide information about $(s-\bar{s})\,D_s^{h^+-h^-}$ as well. It has been suggested that a relatively big $s-\bar{s}$ difference could be generated in NNLO perturbative QCD [17].

If $h=\pi^\pm$, eq.(7) implies that $\sigma_{pp}^{\pi^+-\pi^-}$ is expressed solely in terms of $D_u^{\pi^+-\pi^-}$:

$$\begin{aligned} E^h\frac{d\,\sigma_{pp}^{\pi^+-\pi^-}}{d^3P^\pi} &= \frac{1}{\pi}\int dx_a\,dx_b\,\frac{dz}{z}\,[L_u(x_b,t,u)u_V(x_a) - L_d(x_b,t,u)d_V(x_a) + \\ &\quad + L_u(x_a,u,t)u_V(x_b) - L_d(x_a,u,t)d_V(x_b)]\,D_u^{\pi^+-\pi^-}. \qquad (23) \end{aligned}$$

Thus, not only the SIDIS cross sections $\sigma_p^{\pi^+-\pi^-}$ and $\sigma_d^{\pi^+-\pi^-}$, but also the single inclusive proton-proton collisions $\sigma_{pp}^{\pi^+-\pi^-}$ are expressed in terms of the single quantity $D_u^{\pi^+-\pi^-}$ if $D_u^{\pi^+-\pi^-}=-D_d^{\pi^+-\pi^-}$ holds.

If $h=K^\pm$ the difference cross sections will determine only $D_u^{K^+-K^-}$ and $D_d^{K^+-K^-}$, which would test the assumption $D_d^{K^+-K^-}=0$. If $D_d^{K^+-K^-}=0$, then $d\sigma_{pp}^{K^+-K^-}$ will be expressed solely in terms of one fragmentation function, $D_u^{K^+-K^-}$.

Recently, BRAHMS(RHIC) [18] presented data on $\pi^\pm$ and $K^\pm$ production, that might allow to form the above differences with reasonable accuracy.

3 $K^\pm$ and K_s^0 production

If in addition to the charged $K^\pm$ also neutral kaons $K_s^0=(K^0+\bar{K}^0)/\sqrt{2}$ are measured, no new FFs are introduced into the cross-sections. This is a consequence

of SU(2) invariance of the strong interactions. We have:

$$\begin{aligned}
&D_u^{K^++K^--2K_s^0} = -D_d^{K^++K^--2K_s^0} = (D_u - D_d)^{K^++K^-}, \\
&D_s^{K^++K^--2K_s^0} = D_c^{K^++K^--2K_s^0} = D_b^{K^++K^--2K_s^0} = D_g^{K^++K^--2K_s^0} = 0. \qquad (24)
\end{aligned}$$

We shall show that the combination

$$\sigma^{K^++K^--2K_s^0} \equiv \sigma^{K^+} + \sigma^{K^-} - 2\sigma^{K_s^0} \qquad (25)$$

in the three types of semi-inclusive processes

$$e^+ + e^- \to K + X, \qquad K = K^\pm, K_s^0 \qquad (26)$$

$$e + N \to e + K + X, \qquad N = p, d, \qquad K = K^\pm, K_s^0, \qquad (27)$$

$$p + p \to K + X, \qquad K = K^\pm, K_s^0 \qquad (28)$$

always measures only one NS combination of FFs, namely $(D_u - D_d)^{K^++K^-}$. This result relies only on SU(2) invariance for the kaons and does not involve *any* assumptions about PD's or FFs; it holds in any order in QCD. We shall consider the three processes separately.

Semi-inclusive kaon production in e^+e^- and eN scattering was considered earlier in [10]. For completeness we qoute the results.

3.1 $e^+ + e^- \to K + X$

For the z-distribution in $e^+e^- \to (\gamma, Z) \to K + X$ we have [4]:

$$d\sigma_{e^+e^-}^{K^++K^--2K_s^0}(z, Q^2) = 6\,\sigma_0\,(\hat{e}_u^2 - \hat{e}_d^2)(1 + \frac{\alpha_s}{2\pi}\,C_q \otimes)\,D_{u-d}^{K^++K^-}(z, Q^2) \qquad (29)$$

Here $\sigma_0 = 4\pi\alpha_{em}^2/3\,s$ and

$$\hat{e_q}^2(s) = e_q^2 - 2e_q\,v_e\,v_q\,\Re e\,h_Z + (v_e^2 + a_e^2)\,\left[(v_q)^2 + (a_q)^2\right]\,|h_Z|^2, \qquad (30)$$

where $h_Z = [s/(s - m_Z^2 + im_Z\Gamma_Z)]/\sin^2 2\theta_W$, e_q is the charge of the quark q in units of the proton charge, and, as usual,

$$\begin{aligned}
v_e &= -1/2 + 2\sin^2\theta_W, \quad a_e = -1/2, \\
v_q &= I_3^q - 2e_q\sin^2\theta_W, \quad a_q = I_3^q, \quad I_3^u = 1/2, \quad I_3^d = -1/2. \qquad (31)
\end{aligned}$$

z is the fraction of the momentum of the fragmenting parton transferred to the hadron h: $z = 2(P^h.q)/q^2 = E^h/E$, where E^h and E are the CM energies of the final hadron h and the initial lepton, and $\sqrt{s} = 2E$.

[4] A misprint in the corresponding formula in [10] has been corrected here.

3.2 $eN \to e + K + X$

The cross-sections are given by

$$d\sigma_p^{K^++K^--2K_s^0} = \frac{1}{9}[(4\tilde{u} - \tilde{d}) \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) + \frac{\alpha_s}{2\pi} g \otimes C_{gq}] \otimes \ D_{u-d}^{K^++K^-} \tag{32}$$

$$d\sigma_d^{K^++K^--2K_s^0} = \frac{1}{3}[(\tilde{u} + \tilde{d}) \otimes (1 + \frac{\alpha_s}{2\pi} C_{qq}) + 2\frac{\alpha_s}{2\pi} g \otimes C_{gq}] \otimes \ D_{u-d}^{K^++K^-} \tag{33}$$

3.3 $pp \to K + X$

From Eq. (14), using (24), for $pp \to K + X$, $K = K^\pm, K_s^0$ we obtain:

$$\begin{aligned} E^K \frac{d\sigma_{pp}^{K^++K^--2K_s^0}}{d^3 P^K} &= \frac{1}{\pi} \sum_{a,b} \int dx_a \int dx_b \int \frac{dz}{z} \times \\ &\times f_a^A(x_a) f_b^B(x_b) \left[d\sigma_{ab}^{uX} + d\sigma_{ab}^{\bar{u}X} - d\sigma_{ab}^{dX} - d\sigma_{ab}^{\bar{d}X} \right] D_{u-d}^{K^++K^-}(z) \end{aligned} \tag{34}$$

Here the sum over a, b is over all partons.

It is remarkable that all three processes measure the same NS $D_{u-d}^{K^++K^-}$. This implies, in particular, that even if one does not know the combination $D_{u-d}^{K^++K^-}$, it should be possible to fit the data on all four processes solely with one NS fragmentation function, whose evolution will not introduce any other FFs.

Written in detail Eq. (34) reads:

$$\begin{aligned} E^K \frac{d\sigma_{pp}^{K^++K^--2K_s^0}}{d^3 P^K} &= \frac{1}{\pi} \int dx_a \int dx_b \int \frac{dz}{z} \times \\ &\times \Big\{ \Big[\tilde{u}(x_a)[\tilde{d}(x_b) + \tilde{s}(x_b)] - \tilde{d}(x_a)[\tilde{u}(x_b) + \tilde{s}(x_b)] \Big] \hat{\sigma}_{qq'}^{qX}(s,t,u) + \\ &+ 2 \Big[u(x_a)u(x_b) + \bar{u}(x_a)\bar{u}(x_b) - [d(x_a)d(x_b) + \bar{d}(x_a)\bar{d}(x_b)] \Big] \hat{\sigma}_{qq}^{qX}(s,t,u) + \\ &+ \Big[d(x_a)\bar{d}(x_b) - u(x_a)\bar{u}(x_b) \Big] \Big[2\hat{\sigma}_{q\bar{q}}^{q'X}(s,t,u) - \hat{\sigma}_{q\bar{q}}^{(q+\bar{q})X}(s,t,u) \Big] + \\ &+ [\tilde{u}(x_a) - \tilde{d}(x_a)] g(x_b) \Big[\hat{\sigma}_{qg}^{(q+\bar{q}-2q')X}(s,t,u) \Big] \\ &+ [(x_a \leftrightarrow x_b), (t \leftrightarrow u)] \Big\} D_{u-d}^{K^++K^-}(z). \end{aligned} \tag{35}$$

Note that only 8 inclusive processes (5 in LO) contribute. This result is readily obtained using the symmetry properties of the partonic cross sections and Eq. (24).

The BRAHMS data on $K^\pm$-production, combined with the data on K_s^0-production from STAR(RHIC) [19] may allow to form $\sigma_{pp}^{K^++K^--2K_s^0}$ with reasonable accuracy..

In this section we have presented four independent measurements, Eqs. (26), (27) and (28), that determine in a model independent way the NS combination of the

kaon FF $D_{u-d}^{K^++K^-}$. As these expressions are model independent, it would be interesting to compare the resulting FF to the existing parametrizations extracted from e^+e^- data, which were obtained with various assumptions. In addition, Eqs. (29)-(33) allow one to compare the FF obtained in e^+e^- at rather high $Q^2 \simeq m_Z^2$, Eq. (29), with those from SIDIS at quite low Q^2, Eqs. (32) – (33). Comparing with an extraction based on Eq. (34) would provide a challenging test of the universality of the FFs.

Note that the analogous combination for pions – $\sigma^{\pi^++\pi^--2\pi^0}$, for all three types of processes is identically zero with the usually used assumptions $D_c^{\pi^++\pi^-} = 2D_c^{\pi^0}$.

4 Conclusions

Three types of experiments involving high energy collisions of elementary particles with unpolarized beams have been studied: $e^+e^- \to h + X$, SIDIS $eN \to e + h + X$ ($N = p$ and d) and $pp \to h + X$. Based only on factorization and C-invariance of the strong interactions, and without any assumptions about PDFs and FFs, we show that in any order in QCD the difference cross sections $\sigma^{h^+} - \sigma^{h^-}$ for SIDIS and for $pp \to h + X$ are expressed in terms of the same non-singlet FFs $D_{u,d,s}^{h^+-h^-}$. If in addition to charged kaons $K^\pm$, also the neutral K_s^0 can be measured, then SU(2) invariance implies that for all three types of process the combination $\sigma^{K^++K^--K_s^0}$ is expressed solely in terms of one non-singlet combination $(D_u - D_d)^{K^++K^-}$. These measurements do not provide full information about the FFs, but only part of it, which however is model independent and correct in any order in QCD. This allows to test both the existing parametrizations and some of the usually made assumptions. They also provide a test of Q^2-evolution and factorization.

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